Comprehensive Oxford Mathematics and Physics Online School (COMPOS)

Year 10

Physics Assignment 02

DC Circuits

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Due 17th November, 2024

This is the second Physics assignment from COMPOS for Y10. The assignment goes into the topic in more detail than you have done in school. There are links to online videos which we encourage you to watch. You are free to do your own reading around this topic also.

This assignment is designed to stretch you and no student is expected to complete all questions on the first attempt. Some of the problems are hard, but do not let this discourage you. Give each problem a go, and skip to the next one if you are stuck. The questions in each section are arranged in the order of increasing complexity, so you should try all sections.

Very similar problems will be discussed in the webinars so think of the questions you would like to ask. Please submit what you have by the deadline, you are allowed to submit extra work after the review in tutorials. We hope that eventually you will be able to solve most of the problems. Good luck!

Total 47 marks.

1 Basic Ideas

You should be familiar with the basic variables and components of DC circuits. Let us give you a brief reminder. A short overview can be found in this Mr Chernov video.

Charge, Q, is measured in Coulombs (C) and can be positive or negative; it is typically carried by subatomic particles (e.g. electrons) and is quantised in units of the electron's charge $e = 1.602 \times 10^{-19}$ C. Charge is a conserved quantity — we can't create or destroy it.

Current, I, is the rate of flow of charge, i.e. the ratio $I = \dfrac{\Delta Q}{\Delta t}$, where ΔQ is the amount of charge that flows through a cross-section of a wire during the time interval Δt. The current is measured in Amperes (A). If 1 C of charge flows through a wire in a time of 1 s then the current is $I = 1\,\text{A} = 1$ C/s.

Potential difference or Voltage, V_{AB}, is the energy lost or gained by one Coulomb of electric charge[1] between points A and B. Alternatively one can say that the voltage is the difference in the potential energy of one Coulomb of charge between these points. Voltage is measured in Volts (V), $1\,\mathrm{V} = 1$ J/C. A battery of voltage V powering a circuit does work $\Delta E = V\Delta Q$ on a charge ΔQ as it travels from one terminal of the battery to the other. For example, if the voltage on the battery is 12 Volts, and it pushes 2 Coulombs of charge, it does $\Delta E = V\Delta Q = 12\mathrm{V} \times 2\mathrm{C} = 24$ Joules of work pushing the charge. We can also say that the charge *gains* 24 Joules of energy.

Power, P, is the work ΔE done per unit time Δt when driving an electric charge between two points. It is measured in Watts (W) where $1\mathrm{W} = 1$ J/s. Combining the equations written so far, we find

$$P = \frac{\Delta E}{\Delta t} = \frac{V\Delta Q}{\Delta t} = VI.$$

Example 1. A bike light is rated at $P = 1\,\mathrm{W}$ and runs off a $V = 3\,\mathrm{V}$ battery. In $t = 1$ minute how much charge flows through the bulb? If the battery stores the energy $E = 36\,\mathrm{kJ}$ how long can it power the bulb?

Solution: The current equals $I = P/V$ and the charge flown in time $t = 60\,\mathrm{s}$ is $Q = It = 20\,\mathrm{C}$. The lifetime of the battery is $t = E/P = 36\,000\,\mathrm{s} = 10$ hours.

The capacity of a battery is sometimes measured in Ampere-hours (Ah): the number of hours it can maintain a current of 1 Ampere.

Example 2. Find the capacity of the battery from Example 1.

Solution: The battery produces current $I = P/V$ during time $t = E/P$. Hence its capacity is $It = E/V = 12000$ A·s= 3.33 Ah.

We can see that two batteries of the same capacity (in terms of Ah) but different voltages store proportionally different energies.

Problem 1 (1 marks, Isaac Physics). For how long can a 1000 mAh AAA cell power:

a) a 0.25 A filament bulb in a torch?

b) a 50 mA LED in a modern torch?

c) a remote control drawing 0.5 mA?

[1] Energy is gained by electrons due to the *electric field* doing work on them. We will study the electric field and its properties in COMPOS Y12.

2 Ohm's law

Many circuit components obey *Ohm's Law*: the current flowing through a component is directly proportional to the voltage between its terminals:

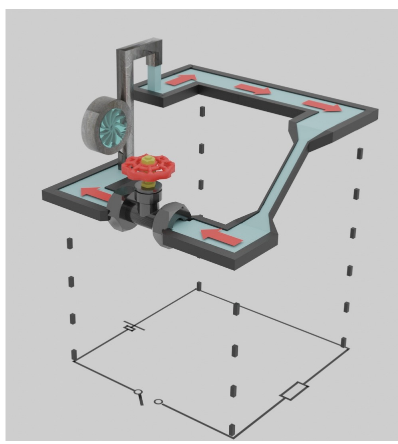
$$I = \frac{V}{R}.$$

The inverse constant of proportionality, R, is known as the *resistance* of the component. It is measured in Ohms: $1\Omega = 1\text{V}/1\text{A}$.

Problem 2 (2 marks). Find the potential difference between the ends of a conductor of resistance $R = 10\,\Omega$ if in $t = 5$ min a charge $q = 120$ C flows through it. How much electric energy will be transferred in the process?

Water analogy[2]

A useful way to understand electric circuits involves imagining a system of ducts with flowing water. In this model the water represents the charge, the rate of flow represents the current, and the change in height (change in the gravitational potential of the water) represents the potential difference (voltage).



Consider a simple water circuit. The pump raises the gravitational potential of the water. The gravitational potential energy transfers to kinetic and sound energy until it eventually dissipates as heat while the water is flowing through a narrow canal. The water then returns to the pump to get more potential energy. In an equivalent electric circuit, the cell or battery raises the electric potential of electrons, these electrons dissipate energy as heat in the resistor. The electrons return to the cell to get more energy.

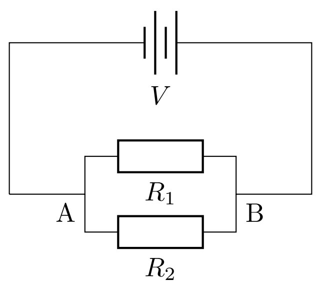
The higher the elevation at the pump, the more water will flow through the canal per second. Similarly, the higher the voltage of the battery, the more current flows through the resistor. The narrower the canal (higher the resistance), the lower the flow (current).

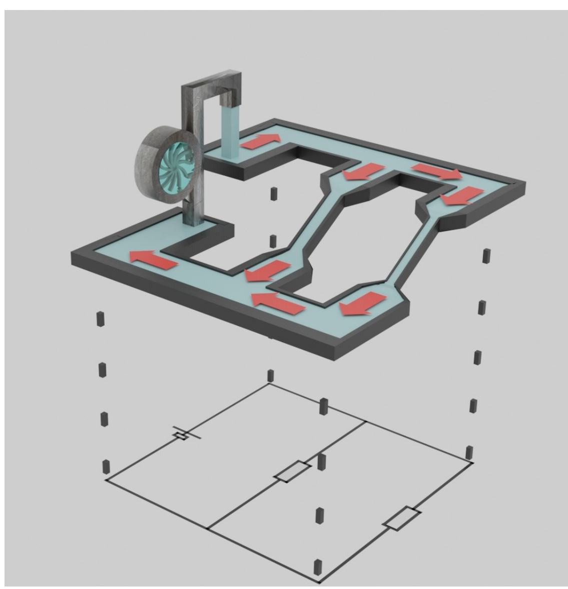
[2] 3D models made by Joe Tarr (COMPOS student).

The wires connecting the battery terminals to the resistor have resistance, too. However, this resistance is very small, so the potential difference between the ends of the wire is negligible. In our analogy, the wires can be thought of as smooth and wide ducts, so a significant amount of water will flow through them even if they are tilted at a very small angle.

3 Resistors in parallel and in series

Example 3. Find the current through the circuit below (*resistors in parallel*).





Solution. Let us first visualize the situation using the water analogy. The water supplied by the pump splits into two canals, and then recombines to enter the pump again. The flow through the pump is the sum of the flows through the two canals. Similarly, the total current in the circuit is the sum of the currents of the two resistors, $I_T = I_1 + I_2$. Applying Ohm's law to each of the resistors, we have $I_1 = V/R_1$ and $I_2 = V/R_2$. Hence $I_T = V\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$.

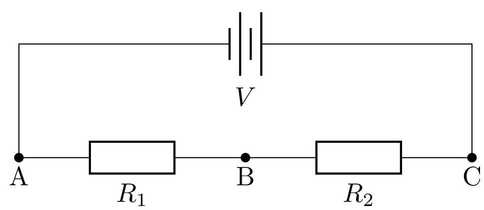
One can rewrite the above result in the form of Ohm's law: $I_T = V\dfrac{1}{R_T}$, where

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

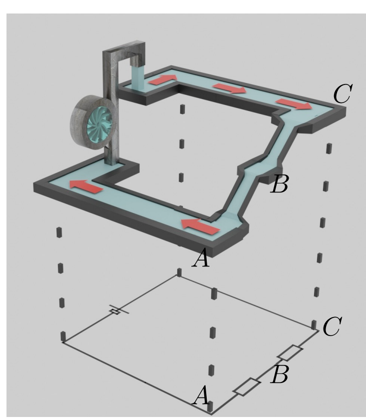
is the *equivalent resistance*. In other words, we can think of a parallel pair of resistors as a single circuit element with resistance R_T.

We can see that the resistance of a parallel pair is smaller than either of the two resistors. This is not surprising: adding another resistor in parallel to an existing one will create a new channel for the current to flow, and hence increase the current.

Example 4. Find the current through the circuit below (*resistors in series*).



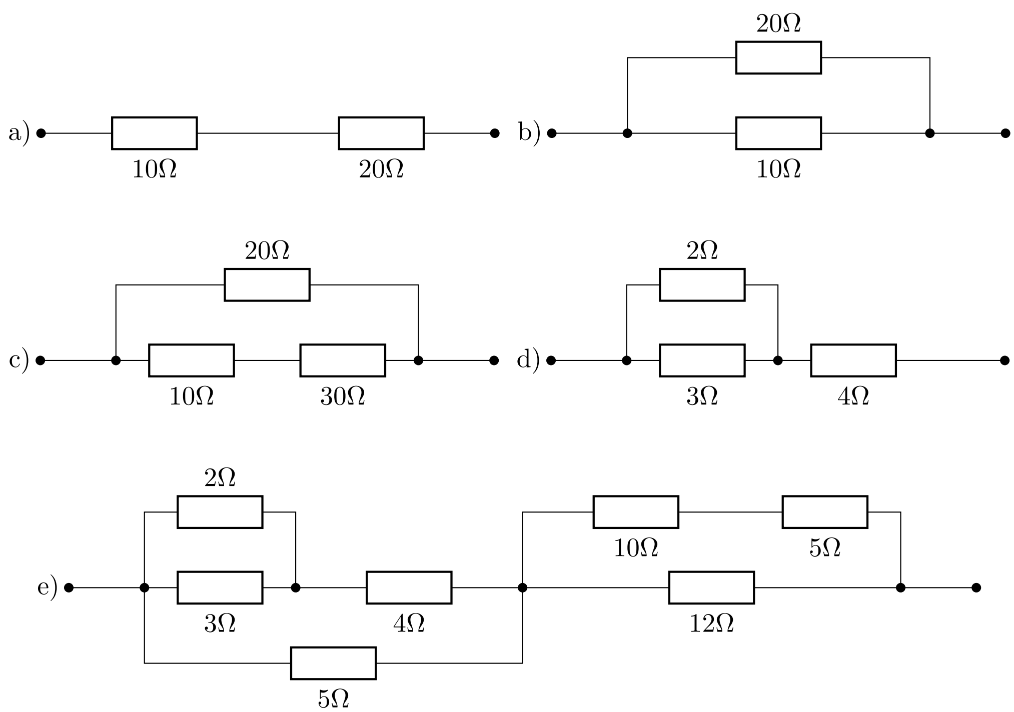
Solution. As we can understand from the water illustration below, the potential difference V_{AC} between points A and C is equal to the battery voltage V. At the same time, it equals $V_{AC} = V_{AB} + V_{BC}$. The circuit does not branch, so the current, I, is the same everywhere. From Ohm's law we have $V_{AB} = IR_1$ and $V_{BC} = IR_2$. Putting everything together, we find $I = \dfrac{V}{R_1 + R_2}$.



This means that the equivalent resistance of the two resistors in series is simply the sum of their resistances: $R_T = R_1 + R_2$.

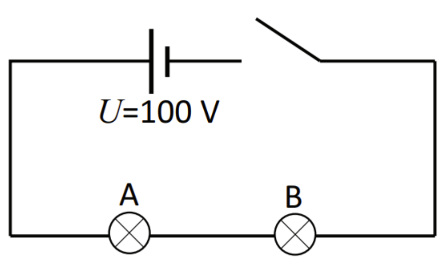
The above example shows some limitations of our water analogy to electric circuit. In a system of water ducts, the altitude of each point is determined by the landscape; it does not depend on the water flow. In an electric circuit, in contrast, the potential at point B "automatically" adjusts itself in such a way that the voltages on both resistors obey Ohm's law.

Problem 3 (6 marks). Find the resistance of the following combinations of resistors (give your answer in exact form as fractions):



Problem 4 (2 marks). An ideal voltmeter must have infinite resistance and should be connected parallel to the circuit element whose potential difference needs to be measurement. In contrast, an ammeter needs to be connected in series with the circuit element and should ideally have zero resistance. Explain why it is the case, using $R_T = R_1 + R_2$ and $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$.

Problem 5* (PAT 2018, 4 marks).

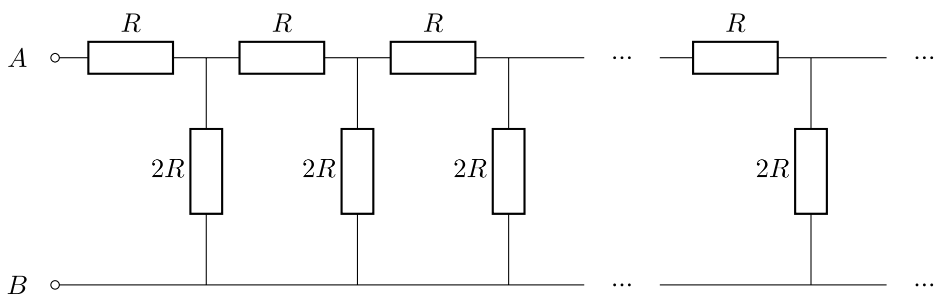


Consider the above circuit. The filament lamp A consumes an electrical power $P_A = 100\,\mathrm{W}$ when it alone is connected to a voltage of $U = 100\,\mathrm{V}$. Filament lamp B consumes a power $P_B = 20\,\mathrm{W}$ when it alone is connected to a voltage of $U = 100\,\mathrm{V}$.

a) When the switch is closed, which lamp will be brighter and why? Find the ratio of the levels of brightness of the two lamps assuming their resistances are constant. Assume further that the brightness of a lamp is proportional to the power it consumes.

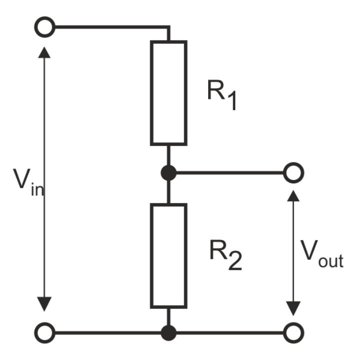
b) How would your answer change if the lamps were wired in parallel rather than series?

Problem 6* (5 marks). Find the total resistance between points A and B in the infinite chain of resistors (see COMPOS Webinar 18 for a similar problem):



4 Potential Dividers

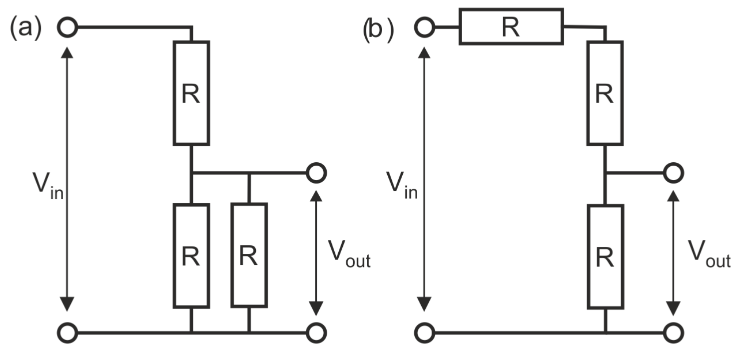
Example 5. For the circuit below give an expression for V_{out} in terms of V_{in}.



Solution. We have two resistors in series. As we know from Example 4, the current through the circuit is $I = V_{\text{in}}/(R_1 + R_2)$ and hence the potential difference on R_2 is $V_{\text{out}} = IR_2 = \dfrac{R_2}{R_1 + R_2} V_{\text{in}}$.

This is a *potential divider* — the voltage across R_2 is given by the ratio of the resistances. This fraction can be controlled, for example, by a rheostat, resulting in a variable voltage source.

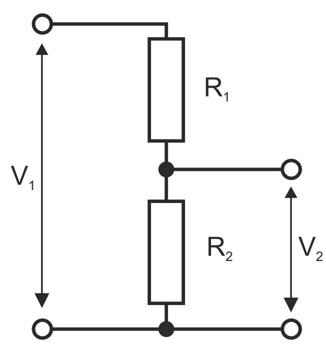
Problem 7 (2 marks). For each of the circuits below give an expression for V_{out} in terms of V_{in}.



Problem 8* (4 marks). For the potential divider (below), where $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 4\,\mathrm{k\Omega}$ and $V_1 = 5\,\mathrm{V}$ find the voltage drop V_2 across R_2 .

a) When a load resistor R_L is fitted in parallel with R_2, what minimum value must R_L have in order not to change V_2 by more than 5%?

b) For R_L at this minimum value, find the electric powers transferred on each of the three resistors.



This problem demonstrates a shortcoming of the voltage divider as a variable voltage source. To avoid disrupting the output voltage, the load resistance must be much less than the two resistors making up the divider. But that means that most of the power will be wasted on these two resistances rather than the load. Therefore in practice other solutions are used to construct variable voltage sources — for example, variable transformers.

5 More Complex Circuits*

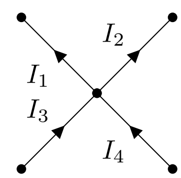
5.1 Kirchhoff's Laws

We have seen that identifying sets of resistors that are in parallel or in series often allow one to simplify a circuit. However, this solution is far from universal. Even amongst resistor networks there are those that cannot be simplified in this manner — see, for example, Problem 10 below. But what if the circuit, in addition to resistors, contains other elements?

We will now discuss a powerful technique that is helpful for solving a broad range of circuits. It is based on two simple rules known as *Kirchhoff's laws.* They can be readily understood from the water analogy.

Kirchhoff's Current Law (KCL): Counting the outgoing currents as positive, and incoming as negative, the sum of all currents at a node is zero: $\sum_{\text{node}} I_n = 0$.

For example, for the circuit node below, we have $I_3 + I_4 = I_1 + I_2$ thus $I_1 + I_2 - I_3 - I_4 = 0$.



Kirchhoff's voltage law (KVL): around *any* closed loop the sum of voltage drops is equal to the sum of voltage raises.

In the language of our water circuits, this is simply the fact that, if you walk along a closed path (i.e. your walk ends at the same place where it started), you will go uphill as much as downhill, no matter which path you take.

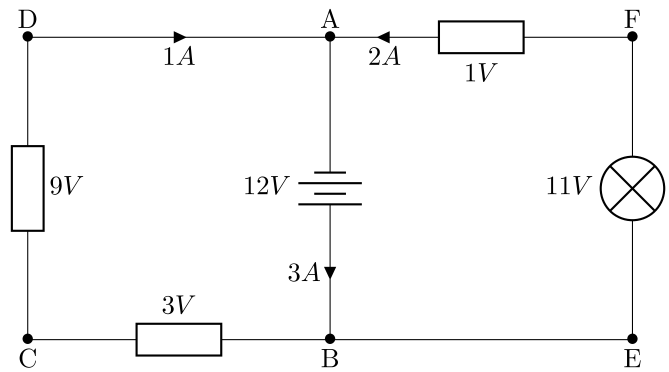
5.2 Examples

Before we proceed, we should tell you two additional conventions.

- *The potential difference between the terminals of an ideal battery is equal to the nominal voltage (electromotive force or EMF) of the battery.* In circuit diagrams, the positive terminal (higher potential) is shown by a longer bar.
- *The current flows in the direction from higher to lower potential ("from plus to minus").* Note that this direction is opposite to the actual movement of negatively charged electrons.

We begin with a simpler example where the currents are already known.

Example 6. Check that the voltages and currents marked in the circuit below are consistent with Kirchhoff's laws.



Solution. There are 3 loops in this circuit, for which KVL can be written: ABCD, ABEF and DFEC.

Going around the loop ABCD clockwise (in the direction of current) voltage rises $= 12V$, Voltage drops $= 3V + 9V = 12V$.

Going around the loop ABEF anticlockwise (in the direction of current): Voltage rises $= 12V$, Voltage drops $= 11V + 1V = 12V$.

Going around the loop DFEC (clockwise) is slightly more complicated. If we go through a resistor in the direction of the current, it is a voltage drop, if we go against the current, it is a voltage rise (it is easy to visualise with the water circuit: if you are going *against* the flow, you are going *uphill*). That is, the potential at point F is higher than that at point A, and that at point E is higher than that at F. Hence we count segments AF abd FE as voltage rises: $= 1V + 11V = 12V$. Segments BC and CD are in the direction of the current, hence counted as voltage drops $= 3V + 9V = 12V$.

We see that KVL is held.

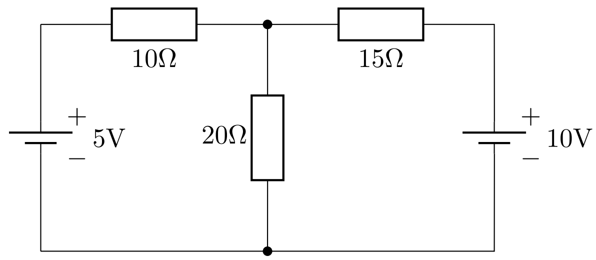
We also write KCL for nodes A and B:

for node A: $2A + 1A - 3A = 0$

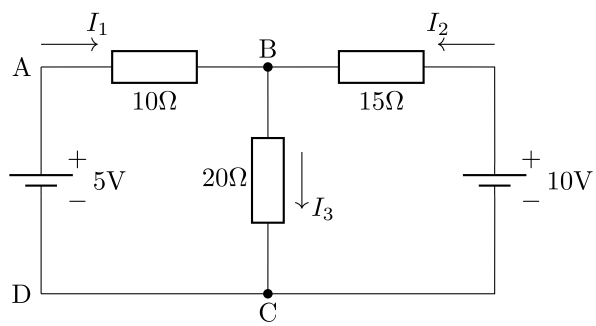
for node B: $3A - 1A - 2A = 0$

To apply Kirchhoff's laws to calculate currents, it is important to know in which direction the currents are flowing. In the problems studied so far, these directions were obvious, but what to do if this is not the case?. The good news is, you can make random assumptions about the current directions. Your calculations will automatically correct you by producing negative currents in those places you guessed the direction incorrectly.

Example 7. For the following circuit, find the currents supplied by both batteries.



Solution. We begin by assigning a variable to the current in every leg of the circuit and making random assumptions about their directions:



Next, we apply KCL to the top node (B) to find:

$$I_3 = I_1 + I_2.$$

Let us now apply KVL to the left internal loop (here going clockwise). We have:

- For the leg AB, the potential difference is $10\Omega \times I_1$. Because the current is assumed flowing from A to B, we assume that A has a higher potential than B, i.e. $V_{AB} = 10\Omega \times I_1$.
- Similarly, for the leg BC, we have $V_{BC} = 20\Omega \times I_3$.
- The leg CD has zero resistance, so $V_{CD} = 0$.
- The leg DA has a battery, so $V_{DA} = -5\text{V}$.

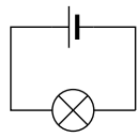
$$-5\,\text{V} + 10\,\Omega \times I_1 + 20\,\Omega \times (I_1 + I_2) = 0. \tag{1}$$

For the right loop, we find (check this!)

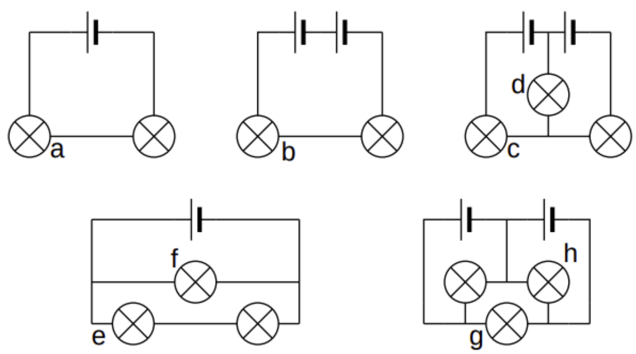
$$-10\,\text{V} + 15\,\Omega \times I_2 + 20\,\Omega \times (I_1 + I_2) = 0. \tag{2}$$

Solving these simultaneous equations gives $I_1 = -\frac{1}{26}\text{A}$, $I_2 = \frac{8}{26}\text{A}$ and $I_3 = I_2 + I_1 = \frac{7}{26}\text{A}$. Because we found I_1 to be negative, we conclude that we guessed its direction incorrectly. (Please note that in A-level and GCSE exams you should give your answer in decimal form, not as fractions.)

Problem 9 (PAT 2006, 3 marks). The diagram below shows a circuit in which the bulb lights up with normal brightness.

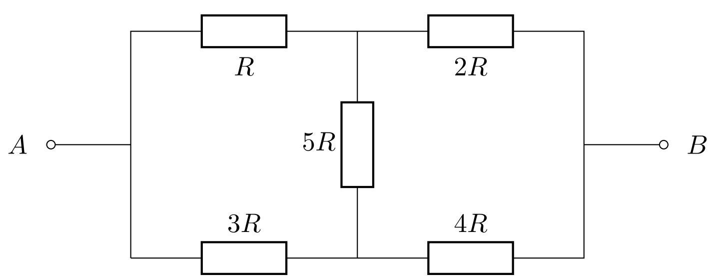


In the circuits below the bulbs and battery cells all have the same specifications as the bulb and cell shown above.



Determine whether the bulbs marked by letters in these circuits are brighter than normal, normal, dimmer than normal, or off.

Problem 10*** (5 marks). Find an expression for total resistance between points A and B in the resistor network below.

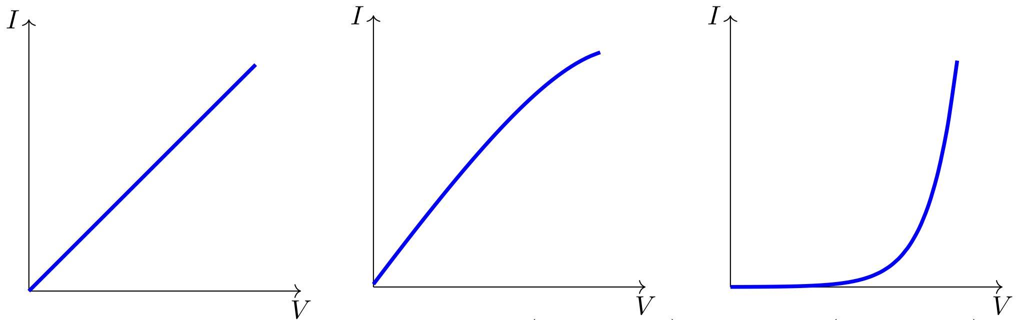


Problem 11* (5 marks). The structure of a cube is soldered by using resistors R for its edges. What is the resistance between the two most distant corners (diametrically opposite)?

Hint: the solution can be greatly simplified by invoking symmetry arguments.

6 Nonlinear components

Circuit elements that do not obey Ohm's law are called *non-linear* or *non-ohmic* because the current is not directly proportional to the voltage, see the graphs below. In this case we can say that the element's resistance, still defined as the voltage-current ratio, is a function of these variables. However, if we know the *current–voltage characteristic (IV-curve)* of the element — that is, how the current through the element depends on the voltage, we can still predict the circuit behaviour.



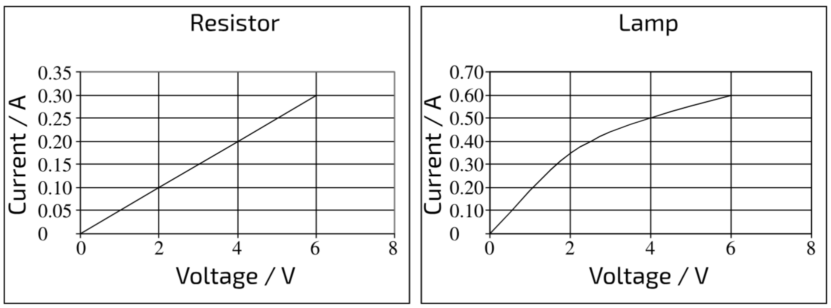
Ohmic Resistor   Filament Lamp (Non-Ohmic)   Diode (Non-Ohmic)

Problem 12 (2 marks). The resistance of a non-linear resistive element is proportional to the square of the current flowing through it: $R = \alpha I^2$, where $\alpha = 0.01\Omega/\mathrm{A}^2$. Find the current if the voltage $V = 5\mathrm{V}$ is applied to this element.

Problem 13 (Isaac Physics, 4 marks). The IV-characteristics of a resistor and a lamp for the range 0-6V are shown below. They form a circuit with a power supply. Find the current and voltage on each component if:

a) they are wired in parallel to a 6V power supply;

b) they are wired in series to a 14V power supply.



Problem 14* (PAT 2007, 2 marks). The current in amperes through a certain type of non-linear resistor is given by $I = 0.05V^3$, where V is the potential difference in volts across the resistor. This resistor is connected in series to a fixed (Ohmic) resistor and a constant voltage source of 9 V is connected across the series combination. What value of resistance should the fixed resistor have so that a current of 0.40 A flows?

7 Optional extra problems

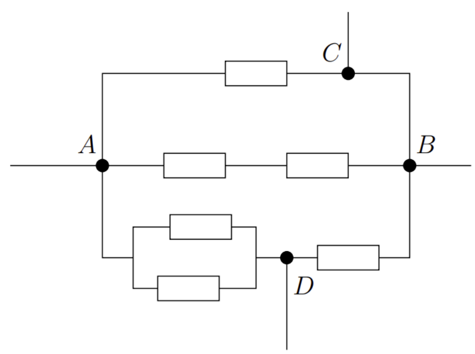
Problem 15 (PAT 2015). Consider the resistor array below. All the resistors are identical, with resistance R.

a) Calculate the total resistance between A and B.

b) If a potential difference V is applied between A and B, calculate the power dissipated in the resistor between B and D.

c) Calculate the total resistance between C and D.

[Leave answers as fractions multiplied by powers of R and V as needed.]



8 Summary

In this assignment you have used these relationships:

- voltage = current × resistance
- voltage = energy transferred ÷ charge
- power = current × voltage
- resistance in series: $R_T = R_1 + R_2 + ... + R_n$
- resistance in parallel: $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + ... + \dfrac{1}{R_n}$

TOTAL MARKS = 47