Comprehensive Oxford Mathematics and Physics Online School (COMPOS)

Year 12

Physics Assignment 01

Review of Kinematics and Newton's Laws

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Due 10 October, 2024

This is the first Physics assignment from COMPOS Y12. This assignment is designed to stretch you and no student is expected to complete all questions on the first attempt. The problems are hard, but do not let this discourage you. Give each problem a go, and skip to the next one if you are stuck. The questions in each section are arranged in the order of increasing complexity, so you should try all sections. Very similar problems will be discussed on webinars every Wednesday at 6pm, so think of the questions you would like to ask. Please submit what you have by the deadline. We hope that eventually you will be able to solve most of the problems. Good luck!

Total 46 marks.

Our first assignment covers the same material as Y10 assignments 3, 4, 5, 6. It will help you get up to speed if you did not participate in Y10 COMPOS. But even if you did, we have a set of new problems for you that we hope you will find both interesting and challenging!

1 Kinematics

1.1 Motion with constant velocity

The main purpose of mechanics is to know the position of an object (*material point*) at any given time. The position is described with respect to a certain *reference frame* by the *position vector (radius-vector)* $\vec{r} = \vec{r}(t)$. This vector is drawn from the *origin* of the reference frame to the current position of the object.

If an object is moving in a straight line with constant speed (i.e with a constant velocity vector $\vec{v}$), its position vector can be described by $\vec{r}(t) = \vec{r}_0 + \vec{v}t$ [where $\vec{r}_0 = \vec{r}(0)$] or in the column vector form:

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} v_x \\ v_y \end{pmatrix} t. \tag{1}$$

Those who studied calculus are aware that velocity is the *derivative* of position[1] with respect to time: $\vec{v} = \dfrac{d\vec{r}(t)}{dt}$.

Before proceeding, please review Examples 1–3 from Y10 Assignment 4.

[1] To take the derivative of a vector by a scalar, differentiate each of its components individually.

Symbolic and numeric answers

The best practice in solving a physics problem is to keep the notation symbolic and obtain the answer as a symbolic expression — like we did in the above example. After that, you can substitute the values into your result and calculate the answer in the numerical form. If you choose to perform the calculation directly with the numbers, you should keep in mind that almost every physical quantity has a unit and always show it, like we did in those example you have seen. It is a *faux pas* to omit the units. If you write, for example, that the time is equal to $t = 300$, your reader will be left to guess if it is 300 nanoseconds, seconds or years that you had in mind!

Problem 1 (3 marks). A drone and an antidrone missile are moving along perpendicular paths in one horizontal plane. The drone passes the intersection point of the two paths at $t = 0$, and the missile passes that point at $t = \tau$, where $\tau = 5$ s. The speed of the drone is $v_1 = 100$ m/s and the speed of the missile is $v_2 = 200$ m/s. Find:

a) the time at which the missile and the drone are closest;

b) the minimal distance between the missile and the drone.

Do not use calculus.

Our second problem is of a kind you may encounter at olympiads, meaning that it requires a degree of inspiration in addition to the knowledge of standard methods. We suggest that you try figuring it out on your own first. If you find yourself struggling, this Nagwa video explains the physical model you should use.

Problem 2* (5 marks). A supersonic plane is flying at a height $H = 3$ km directly above a small village. The residents hear the sound $t = 8$ s after the plane has flown above their heads. Find the speed of the plane. Assume speed of sound is $v_s = 330$ m/s.

1.2 Uniformly accelerated motion

If an object is moving with a constant acceleration $\vec{a}$ the equation of motion becomes:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} v_{0x} \\ v_{0y} \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} a_x \\ a_y \end{pmatrix} t^2. \tag{2}$$

The velocity vector behaves according to

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t \tag{3}$$

Those familiar with calculus should check that the velocity (3) is the derivative of the position (2) and that the acceleration is the derivative of the velocity. Otherwise, you can check out the derivation of these equations on this Khan Academy page.

Please review Example 4 from Y10 Assignment 4.

We begin our study of uniform acceleration with an important special case of one-dimensional motion: when we want to analyze only one component of the position vector — for example, $y(t)$. A physical example is free fall when the horizontal motion component is absent (or not of interest). Orienting the y axis upwards, we find the corresponding component of the acceleration vector to be $a_y = -g = -9.81$ m/s^2, so Eq. (2) becomes $y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$.

Note that throughout this assignment we neglect air resistance.

Example 1. Two balls are thrown in opposite directions along one vertical line with equal speeds. The first ball is thrown upwards from the ground, the second ball it thrown downwards from a height of $h_0 = 90$ m. Find the initial speed u of the balls if they collide at a height of $h = 30$ m. Assume $g = 10\,\text{m/s}^2$.

Solution: Write down the equations of motion of the two balls:

$$y_1(t) = h_0 - ut - \frac{1}{2}gt^2; \quad y_2(t) = 0 + ut - \frac{1}{2}gt^2.$$

When the balls "meet" at time t, $y_1(t) = y_2(t)$, so

$$h_0 - ut - \frac{1}{2}gt^2 = 0 + ut - \frac{1}{2}gt^2 \Rightarrow ut = \frac{h_0}{2}.$$

On the other hand, from the equation of motion for the second ball, we have $h = ut - gt^2/2$. Substituting the above result for ut, we have

$$h = h_0/2 - \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{h_0 - 2h}{g}}.$$

Hence

$$u = \frac{h_0}{2t} = \frac{h_0}{2}\sqrt{\frac{g}{h_0 - 2h}} \approx 26.0\,\text{m/s}.$$

One-dimensional uniformly accelerated motion is characterized by a set of equation referred to as SUVAT [as they relate relate: s (displacement), u (initial velocity), v (final velocity), a (acceleration), and t (time)]. You are probably already familiar with them, but let us list them here for reference:

$$s = ut + \frac{at^2}{2}; \tag{4}$$

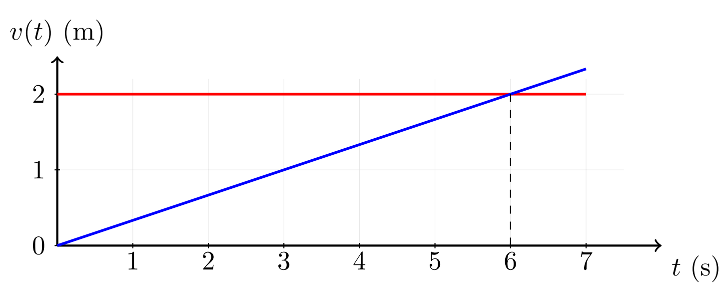
$$v = u + at; \tag{5}$$

$$s = \frac{(v + u)}{2}t; \tag{6}$$

$$2as = v^2 - u^2. \tag{7}$$

The first two of these equations are simply one-dimensional versions of Eqs. (2) (with $\vec{s}(t) = \vec{r}(t) - \vec{r}_0$) and (3). Please refer to Section 3 of Y10 Assignment 3 if you need a reminder on how to derive the remaining two. Please also study Example 4 from this assignment.

Problem 3 (2 marks). Two objects start moving at $t = 0$ from the same point, their velocity-time graphs are given. Find the time when the two object meet again.



Problem 4 (2 marks). A cannon shoots a shell at a hot air balloon located $H = 6$ km directly above it. What is the minimum required acceleration of the shell inside the barrel if the length of the barrel is $l = 2$ m? Assume that the motion inside the barrel is with constant acceleration.

1.3 Projectile motion

Projectile motion is motion in a gravitational field, with both dimensions taken into account. Since the acceleration vector is $\vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$, projectile motion can be described as a combination of

- constant velocity motion in the horizontal dimension;
- free fall in the vertical dimension.

We studied both these kinds of motion previously, but analyzing them together gives rise to interesting new physics.

Please study Example 7 from Y10 Assignment 4 to get an understanding of this motion. A few more examples of projectile motion can be found in these Khan Academy videos: Horizontal projectile, Projectile at an angle and this Mr Chernov video.

Example 2. A stone thrown at an angle $\theta = 30°$ is at the same height h at $t_1 = 3s$ and $t_2 = 5s$. Find the initial velocity u of the stone.

Solution: We can use the same equations of motion as in the above example. Since $\theta = 30°$, we have $\sin\theta = \frac{1}{2}$ and hence $y(t) = \frac{1}{2}ut - \frac{1}{2}gt^2$.

It is given that $y(t_1) = y(t_2)$, so

$$\frac{1}{2}ut_1 - \frac{1}{2}gt_1^2 = \frac{1}{2}ut_2 - \frac{1}{2}gt_2^2 \Rightarrow u = g\frac{t_2^2 - t_1^2}{t_2 - t_1} = g(t_1 + t_2) = 80\,\mathrm{m/s}.$$

Problem 5 (modified PAT 2010, 3 marks). A gun is designed that can launch a projectile, of mass 10 kg, at a speed of 200 m/s. The gun is placed close to a straight, horizontal railway line and aligned such the projectile will land further down the line. A small rail car, of mass 200 kg and travelling at a speed of 100 m/s passes the gun just as it is fired.

a) Assuming the gun and car at the same level, at what angle upwards must the projectile be fired in order that it lands on the rail car?

b) How long does it take for the projectile to reach its maximum altitude?

c) How far is the rail car from the gun when the projectile lands in it?

d) Without considering energy, calculate the projectile's maximum altitude.

2 Dynamics

2.1 Newton's Laws

The previous section — kinematics — dealt with describing *how* objects move. This section - Dynamics, explains *why* they move. Here we will be talking about forces, and we start with a recap of Newton's laws, which should already be familiar to you.

1. There exist reference frames (known as **inertial**) in which an object remains at rest or moves with constant velocity if it is not interacting with other objects.
2. The force acting upon the object is given by its mass times acceleration:
$$\vec{F} = m\vec{a} \tag{8}$$
3. When two objects interact, they exert equal but opposite forces on each other.

You may find these formulations to be somewhat different from those familiar to you. Please review Sections 1 and 2 from Y10 Assignment 5 to understand why. To test yourself, try answering these questions (and discussing them with your tutor).

a) Why can't we rephrase the last clause of Newton's First Law as "...if the net force on an object is zero"?

b) Is Newton's Second Law an experimental observation? What about Newton's Third Law?

c) A horse is pulling a sleigh, which means that the sleigh is also pulling the horse. So why do they move?

d) Baron Münchausen, who once got stuck in a swamp, was famously able to pull himself and his horse out by his own hair. Which laws of physics does this story violate?

2.2 Friction Force

Please study Section 4 from Y10 Assignment 5 and examples therein to learn about static and dynamic friction.

Problem 6 (1 mark). A sleeper train is approaching a station at 144 km/h. How far from the station must the train driver apply the brakes to ensure uniform smooth deceleration so that the passengers don't fall out of their beds? Coefficient of friction of a passenger's body with the bed is $\mu = 0.2$.

Problem 7 (2 marks). A wooden block is positioned on an inclined plane whose inclination angle gradually increases from zero until the block starts moving. The static and dynamic coefficients of friction are $\mu_s = 0.4$ and $\mu_D = 0.3$, respectively.

a) At which inclination angle will the block starts moving?

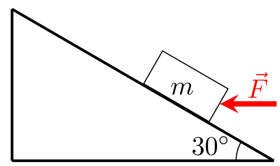
b) What will be its acceleration?

Problem 8 (2 marks). A wooden block is launched up an inclined plane with speed v. Plot the velocity along the inclined plane as a function of time. How much time will elapse before the speed becomes v again? The inclination angle is α, the coefficient of static and dynamic friction $\mu < \tan\alpha$.

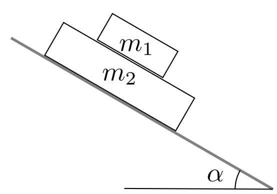
Problem 9 (4 marks). A *horizontal* force $\vec{F}$ is applied to a block of mass $m = 1$ kg placed on an inclined at $\theta = 30°$ plane. The coefficients of static and dynamic friction are $\mu_s = 0.4$ and $\mu = 0.3$, respectively.

a) Find the minimum and maximum F such that the block is at rest.

b) Find F such that the block is moving up the slope with a constant speed.



Problem 10 (4 marks). A block of mass $m_2 = 10$ kg is placed on a rough inclined plane (inclination $\alpha = 30°$), a second block of mass $m_1 = 5$ kg is placed on top of the first one. The coefficient of friction between the two blocks is $\mu_1 = 0.15$ and the coefficient of friction between the bottom block and the slope is $\mu_2 = 0.3$. Find the accelerations of both blocks. At what values of μ_2 will the bottom block remain stationary?



2.3 Tension force

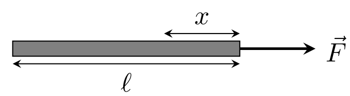
Tension force is the force that we would see if we replaced any fragment of a taut string with a Newton meter. Please study Section 3 from Y10 Assignment 5 and examples therein to learn about it.

Example 3. In the logotype depicted below, each horse pulls with the force $T = 10,000$ N. What is the maximum tensile force that the jeans can withstand according to this demonstration? How would the answer change if one of the horses was replaced by a wall?

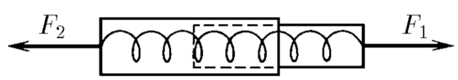
Solution. According to Newton's Third Law, the rope pulls the horse with the same force as the horse pulls the rope, i.e. T. This is the tension of the rope and it is the same everywhere in the rope, as the masses of the rope and jeans are very low. We would have the same tension if one of the horses was replaced by a wall.

Problem 11 (2 marks). Force F pulls one end of a heavy metal bar of length ℓ. Find the tension of the bar at distance x away from the front end of the bar. Neglect gravity and friction.

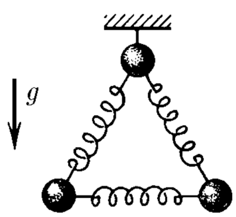
Hint: Consider the bar as a chain of multiple small blocks connected by short strings.



Problem 12 (2 marks). A newton meter consists of two cylinders of masses m_1 and m_2 connected by a light spring. Find the ratio m_1/m_2 if, with the forces F_1 and F_2 applied, respectively, to the first and second cylinder, the meter shows a force of F. Neglect gravity and friction.



Problem 13* (4 marks). A system of three identical balls of mass m, connected by identical massless springs, is suspended on a thread as shown in a diagram. The thread is cut. Find the accelerations of the balls immediately thereafter.

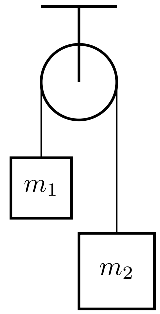


Hint. While the top two springs are taut, the bottom one is compressed. The compression force should be treated similarly to the tension force. Draw free-body diagrams for each of the balls and find the tension/compression forces that are needed to keep each ball at rest before the thread is cut. Immediately after the cutting, these forces remain the same.

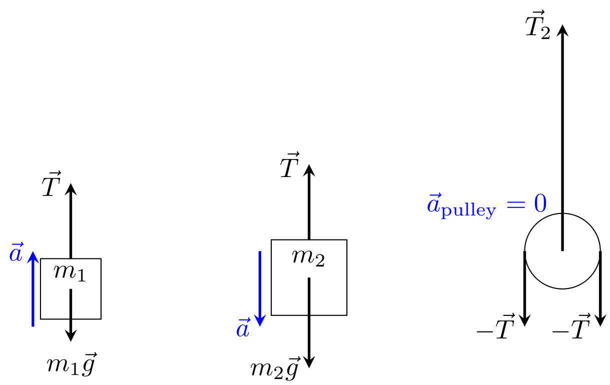
2.4 Pulleys

Our last set of problems is about pulleys. Y10 Assignment 6 is dedicated to this subject and contains many complex settings involving both stationary and mobile pulleys. Here we do not delve that deeply into this subject and consider relatively simple arrangements.

Example 4. Two masses of mass $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a light in-extensible string over a light frictionless pulley. Find the acceleration of each mass.



Solution: Let's draw free-body diagrams for the two masses and the pulley.



Note the following:

- $\vec{T}$ is the tension of the string. It is the same throughout the string, a consequence of both the string and the pulley being massless. Indeed: if the tensions at different points were different, it would result in a net force acting on the part of the string between them, and hence infinite acceleration.
- Because the strings are in-extensible, the accelerations of the two masses are the same in magnitude but opposite in direction. This is an example of so-called *kinematic constraints*. Since m_2 is heavier, we have assumed this block to accelerate downwards (and hence m_1 upwards). In case we do not know the direction of the acceleration, we can assign it randomly (but consistently with kinematic constraints) and solve the equations; if we get a negative answer, this will tell us that the direction is opposite to what we guessed.

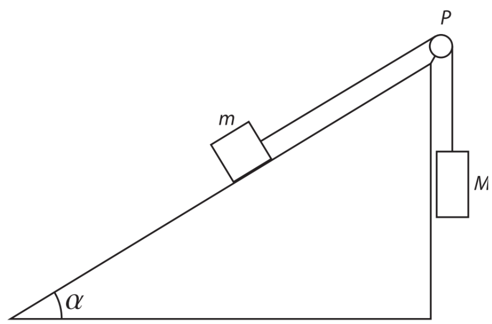
For each of the three objects, we write Newton's 2nd Law. although this law has the vector form, we are only interested in the motion along the vertical axis, hence we write the y projections of the relevant forces and accelerations.

- 1st mass: $m_1 a = T - m_1 g$;
- 2nd mass: $-m_2 a = T - m_2 g$;
- the pulley is not moving, so $a_{\text{pulley}} = 0$, i.e. $2T - T_2 = 0$

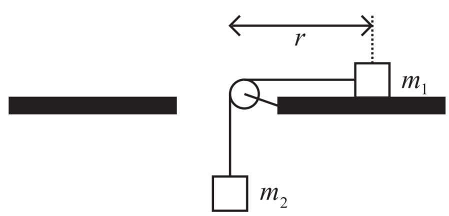
Solving the equations, we find

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = 3.27 \text{ m/s}^2; \quad T = m_1(a+g) = \frac{2m_1 m_2}{m_2 + m_1} g = 13.1 \text{ N}; \quad T_2 = 2T = 26.2 \text{ N}.$$

Problem 14 (PAT 2013, 3 marks). Two masses, m and M, are connected by a massless string of fixed length on a slope inclined at an angle α as sketched in the figure below. The pulley P is massless. Ignoring friction, calculate the acceleration of mass m and the tension of the string. What is the condition for the masses to be stationary?



Problem 15 (PAT 2014, 3 marks). Two masses m_1 and m_2 are connected by a massless, non-extensible string supported by a massless pulley attached to a table with a hole in the middle; see sketch below.



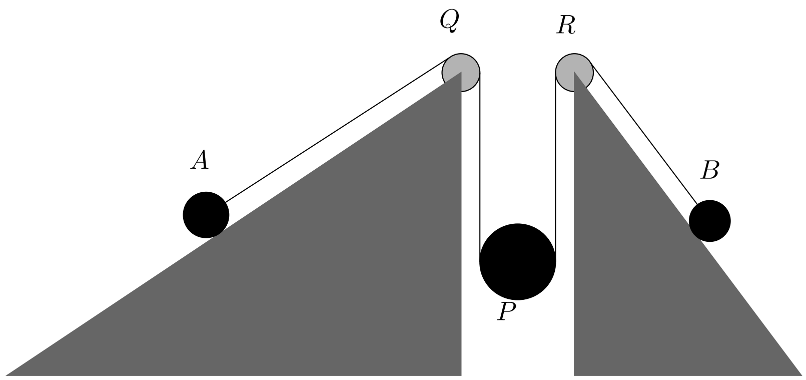
a) Assuming no friction, derive an expression for the acceleration of the masses and for the tension of the string.

Now and for the rest of this question, consider friction acting on the table but not on the pulley. Friction force F_{fr} is proportional to the mass's weight; $F_{\text{fr}} = \mu_s mg$ or $F_{\text{fr}} = \mu_d mg$ depending whether the mass is at rest (μ_s = static friction coefficient) or in motion (μ_d = dynamic friction coefficient). Both coefficients are known.

b) Derive expressions for the acceleration of the masses and for the tension of the string. What condition needs to be satisfied for m_1 to accelerate?

Problem 16 (STEP 2012, 4 marks). The diagram shows two particles, A of mass $5m$ and B of mass $3m$, connected by a light inextensible string which passes over two smooth, light, fixed pulleys, Q and R, and under a smooth pulley P which has mass M and is free to move vertically.

Particles A and B lie on fixed rough planes inclined to the horizontal at angles of $\arctan \frac{7}{24}$ and $\arctan \frac{4}{3}$ respectively. The segments AQ and RB of the string are parallel to their respective planes, and segments QP and PR are vertical. The coefficient of friction between each particle and its plane is μ.



a) Given that the system is in equilibrium, with both A and B on the point of moving up their planes, determine the value of μ and show that $M = 6m$.

b) In the case when $M = 9m$, determine the initial accelerations of A, B and P in terms of g.

Hint: Exampe 1 from Y10 Assignment 6 may be helpful.